AP Calculus AB

Summer Work Packet

2022

<u>Please Read Carefully</u>!

1) You are expected to complete this packet over the summer.

2) You will be held responsible for the knowledge in this packet!

3) There will be a test early in the year that will assess you on much of this content!

4) Before answering the questions on the topic, be sure to read the examples and notes to understand it fully.

5) If you are stuck, there are many great online resources, but Partick JMT has many YouTube videos and so does KhanAcademy.

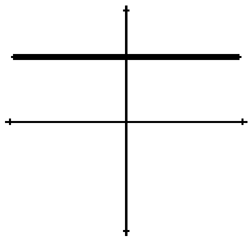
Name: ____________________________________

Toolkit of Functions

Students should know the basic shape of these functions and be able to graph their transformations without the assistance of a calculator.

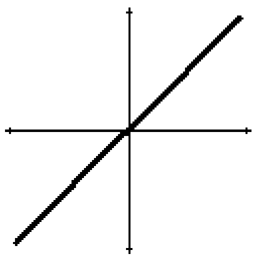
Constant

f(x) = a



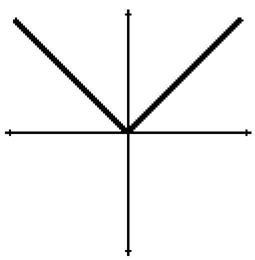
Identity

f(x) = x



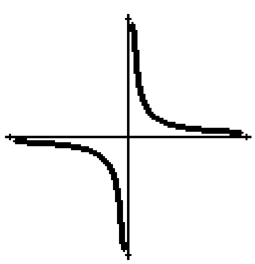
Absolute Value

f(x) = | x |



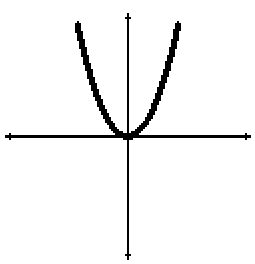
Reciprocal

$f(x) = \dfrac{1}{x}$



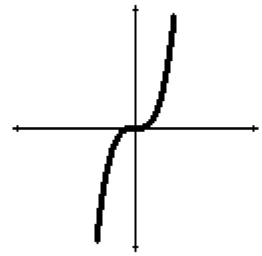
Quadratic

$f(x) = x^2$



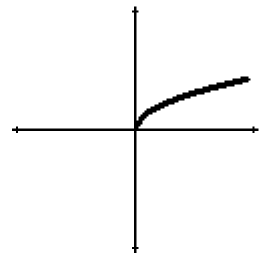
Cubic

$f(x) = x^3$



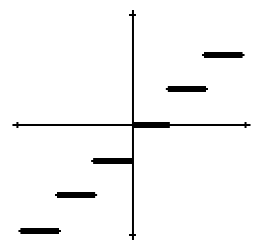
Square Root

$f(x) = \sqrt{x}$



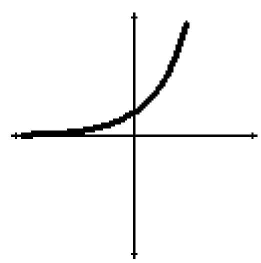
Greatest Integer

f(x) = [x]



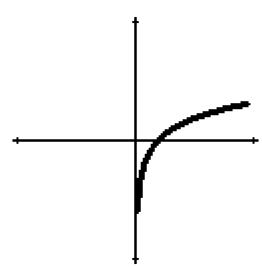
Exponential

$f(x) = a^x$



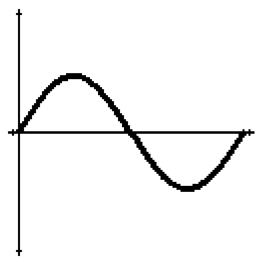
Logarithmic

$f(x) = \ln x$

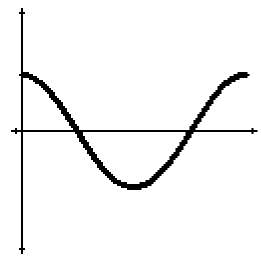


Trig Functions

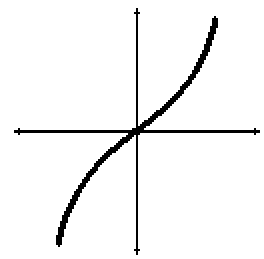
$f(x) = \sin x$



$f(x) = \cos x$



$f(x) = \tan x$



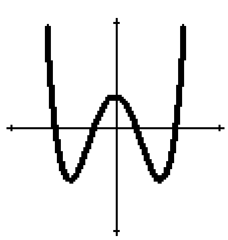
Polynomial Functions:

A function P is called a polynomial if $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$
Where n is a nonnegative integer and the numbers a_0, a_1, a_2, … a_n are constants.

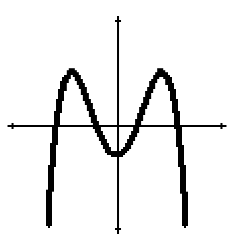
Even degree

Leading coefficient sign

Positive



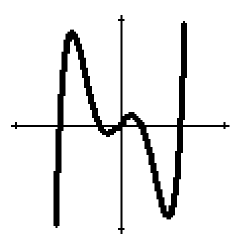
Negative



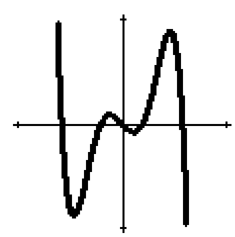
Odd degree

Leading coefficient sign

Positive



Negative



- Number of roots equals the degree of the polynomial.
- Number of x intercepts is less than or equal to the degree.
- Number of "turns" is less than or equal to (degree – 1).

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

1. $f(2) =$ ____________
2. $g(-3) =$ ____________
3. $f(t+1) =$ ____________
4. $f[g(-2)] =$ __________
5. $g[f(m+2)] =$ __________
6. $[f(x)]^2 - 2g(x) =$ ______

Let $f(x) = \sin(2x)$ **Find each exactly.**

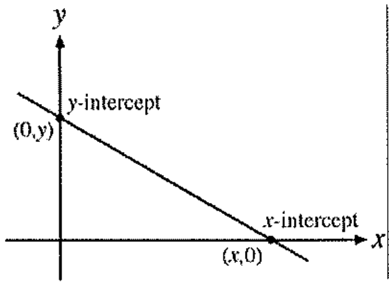
7. $f\left(\frac{\pi}{4}\right) =$ __________
8. $f\left(\frac{2\pi}{3}\right) =$ ______________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

9. $h[f(-2)] =$ _______
10. $f[g(x-1)] =$ _______
11. $g\left[h(x^3)\right] =$ _______

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$, find all intercepts.

$x-\text{int}.\ (Let\ y=0)$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

$y-\text{int}.\ (Let\ x=0)$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y-\text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

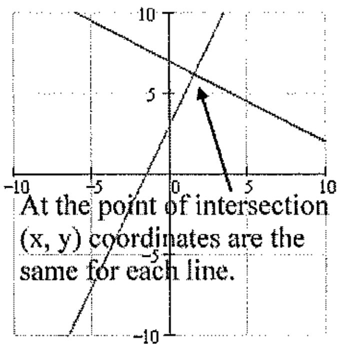
13. $y = x^2 + x - 2$

14. $y = x\sqrt{16 - x^2}$

15. $y^2 = x^3 - 4x$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2^{nd} Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{array}{l} x^2 - y = 3 \\ x - y = 1 \end{array}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x=2$ and $x=-1$ to find y

Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{array}{l} x + y = 8 \\ 4x - y = 7 \end{array}$

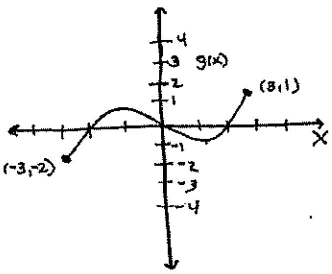
17. $\begin{array}{l} x^2 + y = 6 \\ x + y = 4 \end{array}$

18. $\begin{array}{l} x = 3 - y^2 \\ y = x - 1 \end{array}$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).

The domain is the set of inputs of the function. Input values run along the horizontal axis. The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3. So Domain is [-3, 3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis. The lowest output value of the function is -2. The highest is 1. So the range is [-2, 1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN

For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE

The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x)=x^2-5$

20. $f(x)=-\sqrt{x+3}$

21. $f(x)=3\sin x$

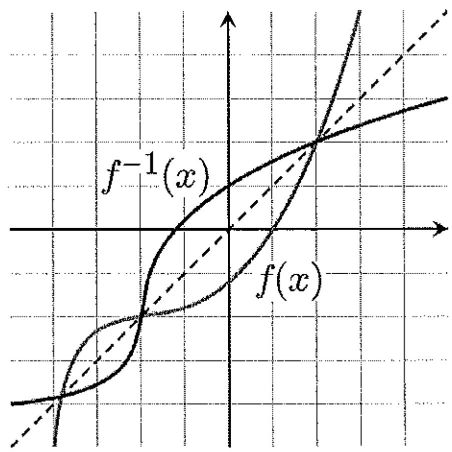
22. $f(x)=\dfrac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value. Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x)=2x+1$

24. $f(x)=\dfrac{x^2}{3}$

25. $g(x)=\dfrac{5}{x-2}$

26. $y=\sqrt{4-x}+1$

27. If the graph of $f(x)$ has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

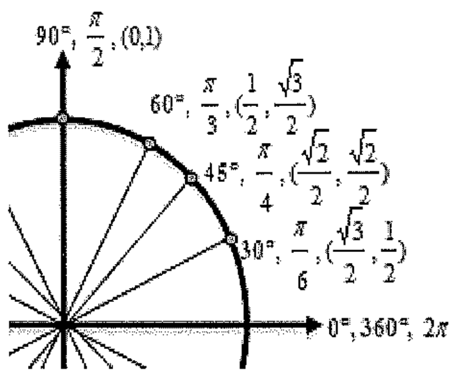
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

34. Find the equation of a line passing through the points (-3, 6) and (1, 2).

35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2} = 1$ $\cos\frac{\pi}{2} = 0$ $\tan\frac{\pi}{2} = und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. *a.*) $\sin\pi$ *b.*) $\cos\frac{3\pi}{2}$ *c.*) $\sin\left(-\frac{\pi}{2}\right)$ *d.*) $\sin\left(\frac{5\pi}{4}\right)$

e.) $\cos\frac{\pi}{4}$ *f.*) $\cos(-\pi)$ g) $\cos\frac{\pi}{3}$ h) $\sin\frac{5\pi}{6}$

i) $\cos\frac{2\pi}{3}$ j) $\tan\frac{\pi}{4}$ k) $\tan\pi$ l) $\tan\frac{\pi}{3}$

m) $\cos\frac{4\pi}{3}$ n) $\sin\frac{11\pi}{6}$ o) $\tan\frac{7\pi}{4}$ p) $\sin\left(-\frac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \le x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$
    **Recall $\sin^2 x = (\sin x)^2$
    **Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x-c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x+c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

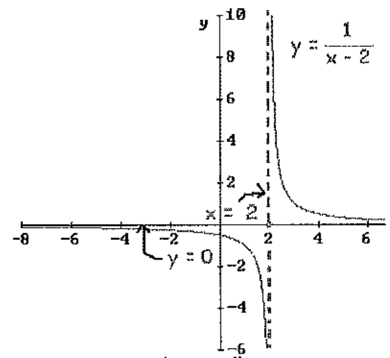
43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

a) $f(x) - 3$ b) $f(x-3)$ c) $2f(x)$ d) $f(x-2)+1$ e) $-f(x)$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \dfrac{1}{x-2}$
Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \dfrac{1}{x^2}$

45. $f(x) = \dfrac{x^2}{x^2 - 4}$

46. $f(x) = \dfrac{2 + x}{x^2(1 - x)}$

47. $f(x) = \dfrac{4 - x}{x^2 - 16}$

48. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

49. $f(x) = \dfrac{5x + 20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \dfrac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \dfrac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \dfrac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \dfrac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \dfrac{x^2 - 2x + 1}{x^3 + x - 7}$

51. $f(x) = \dfrac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52. $f(x) = \dfrac{4x^2}{3x^2 - 7}$

53. $f(x) = \dfrac{(2x-5)^2}{x^2 - x}$

54. $f(x) = \dfrac{-3x + 1}{\sqrt{x^2 + x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$	Get a common base
$2^{2x+2} = 2^{-3x+2}$	Simplify
$2x+2 = -3x+2$	Set exponents equal
$x = 0$	Solve for x

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^x = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.
REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828... or $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$
Therefore $? = 3$
Thus $\log_2 8 = 3$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

60. $\log_2 \frac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln\sqrt{e}$

65. $\ln\frac{1}{e}$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$ $\quad$ $\log_b \frac{x}{y} = \log_b x - \log_b y$ $\quad$ $\log_b x^y = y\log_b x$ $\quad$ $b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \frac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$ $\quad$ 67. $\ln e^3$ $\quad$ 68. $\log_2 8^3$ $\quad$ 69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$ $\quad$ 71. $e^{\ln 8}$ $\quad$ 72. $9\ln e^2$ $\quad$ 73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$ $\quad$ 75. $\log_2 40 - \log_2 5$ $\quad$ 76. $\log_2 \left(\sqrt{2}\right)^5$

<u>EVEN AND ODD FUNCTIONS</u>

Recall:

Even functions *are functions that are symmetric over the y-axis.*

To determine algebraically we find out if $f(x) = f(-x)$

*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

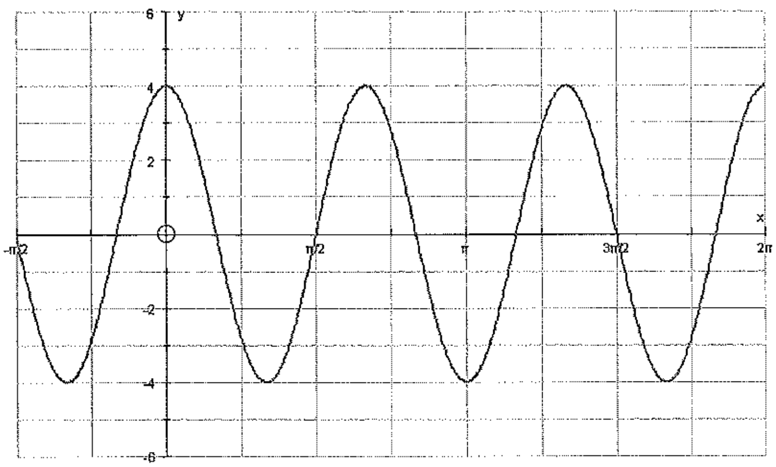
Odd functions *are functions that are symmetric about the origin.*

To determind algebraically we find out if $f(-x) = -f(x)$

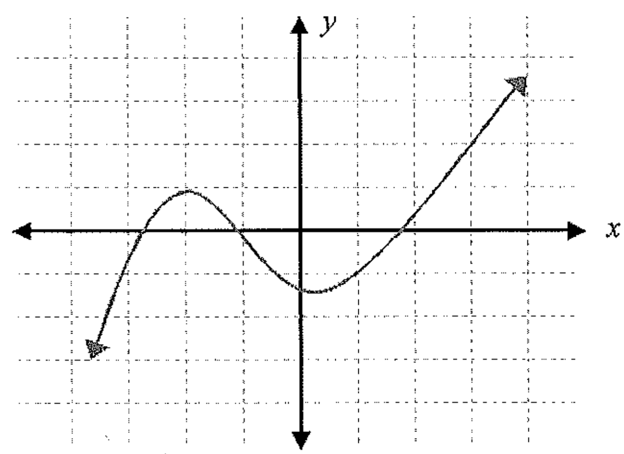
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ____________________



78. ____________________



79. ____________________

$f(x) = 2x^4 - 5x^2$

80. ____________________

$g(x) = x^5 - 3x^3 + x$

81. ____________________

$h(x) = 2x^2 - 5x + 3$

82. ____________________

$j(x) = 2\cos x$

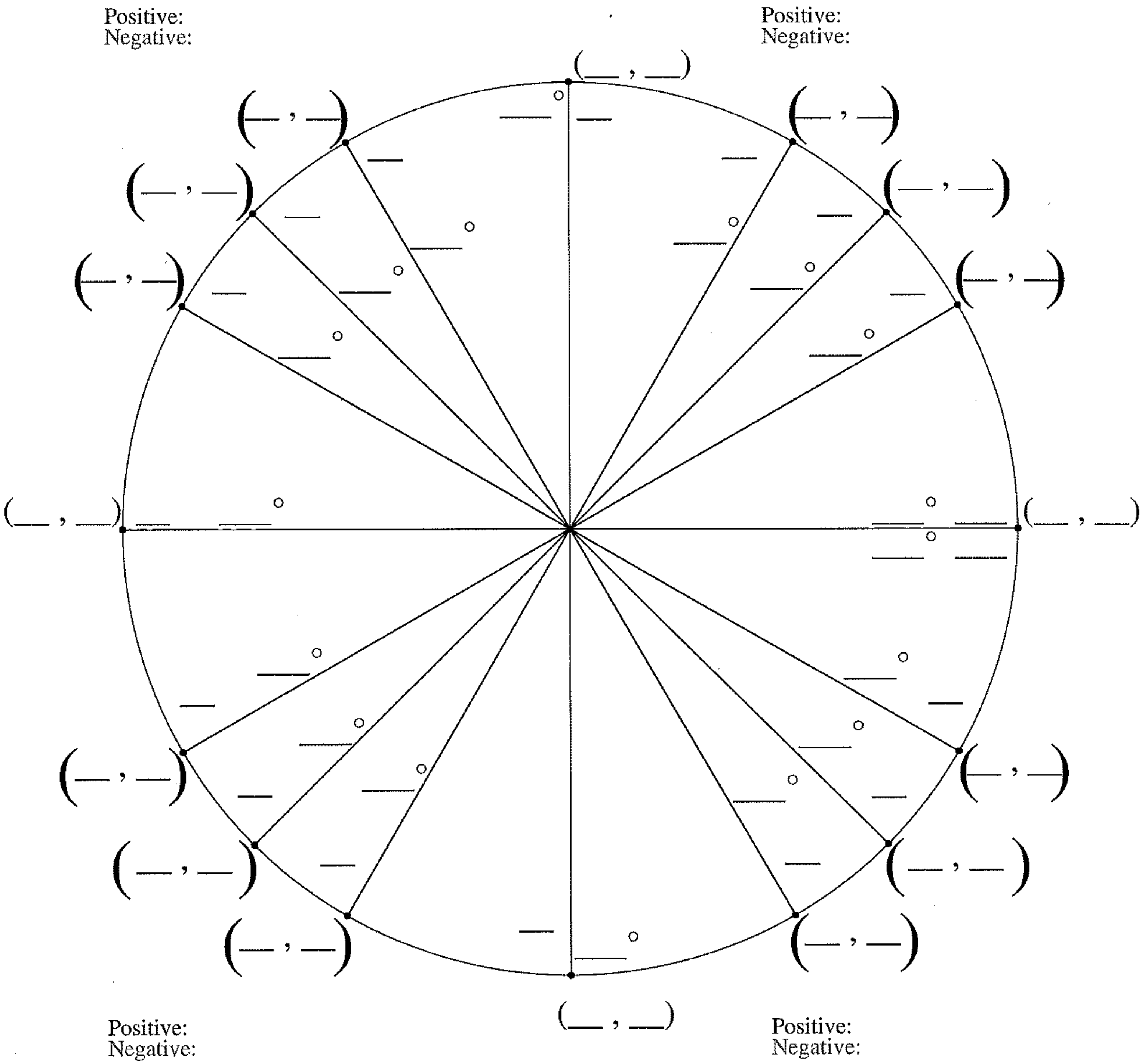
83. ____________________

$k(x) = \sin x + 4$

84. ____________________

$l(x) = \cos x - 3$

Fill in The Unit Circle



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