AP Calculus AB Summer Packet

Within the first few days of your AP Calculus AB course , you will be assessed on the prerequisite skills outlined in this packet. This summer assignment is a review and exploration of key Precalculus skills that is necessary for success in AP Calculus AB as well as future high school math courses. The packet will NOT be graded; however, you are responsible for the material. **The assessment will count as a full test grade in your first quarter average.**

FUNCTIONS

To evaluate a function for a given value, simply plug the value into the function for x.

Recall: $(f \circ g)(x) = f(g(x))$ *OR* $f[g(x)]$ read "***f*** of ***g*** of ***x***" Means to plug the inside function (in this case g(x)) in for x in the outside function (in this case, f(x)).

Example: Given $f(x) = 2x^2 + 1$ *and* $g(x) = x - 4$ find *f(g(x))*.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ *and* $g(x) = 2x^2 - 1$. **Find each.**

1. $f(2) =$ ____________
2. $g(-3) =$ ____________
3. $f(t+1) =$ ____________
4. $f\left[g(-2)\right] =$ ____________
5. $g\left[f(m+2)\right] =$ ____________
6. $\left[f(x)\right]^2 - 2g(x) =$ ______

Let $f(x) = \sin(2x)$ **Find each exactly.**

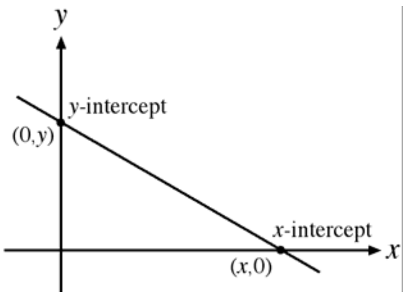
7. $f\left(\frac{\pi}{4}\right) =$ ____________
8. $f\left(\frac{2\pi}{3}\right) =$ ____________

Let $f(x) = x^2$, $g(x) = 2x + 5$, *and* $h(x) = x^2 - 1$. **Find each.**

9. $h\left[f(-2)\right] =$ _______
10. $f\left[g(x-1)\right] =$ _______
11. $g\left[h(x^3)\right] =$ _______

INTERCEPTS OF A GRAPH

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.



Example: Given the function $y = x^2 - 2x - 3$**,** find all intercepts.

$\underline{x - \text{int. } (Let\ y = 0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x - \text{intercepts } (-1,0)\ and\ (3,0)$

$\underline{y - \text{int. } (Let\ x = 0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y - \text{intercept } (0,-3)$

Find the x and y intercepts for each.

12. $y = 2x - 5$

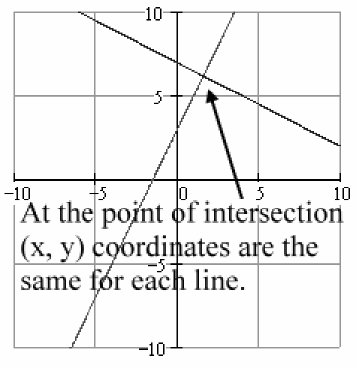
13. $y = x^2 + x - 2$

14. $y = x\sqrt{16 - x^2}$

15. $y^2 = x^3 - 4x$

POINTS OF INTERSECTION

Use substitution or elimination method to solve the system of equations.
Remember: You are finding a POINT OF INTERSECTION so your answer is an ordered pair.



CALCULATOR TIP

Remember you can use your calculator to verify your answers below. Graph the two lines then go to CALC (2^{nd} Trace) and hit INTERSECT.

Example: Find all points of intersection of $\begin{aligned} x^2 - y &= 3 \\ x - y &= 1 \end{aligned}$

ELIMINATION METHOD

Subtract to eliminate y

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

Plug in $x = 2$ and $x = -1$ to find y

Points of Intersection: $(2,1)$ and $(-1,-2)$

SUBSTITUTION METHOD

Solve one equation for one variable.

$y = x^2 - 3$

$y = x - 1$

Therefore by substitution $x^2 - 3 = x - 1$

$x^2 - x - 2 = 0$

From here it is the same as the other example

Find the point(s) of intersection of the graphs for the given equations.

16. $\begin{aligned} x + y &= 8 \\ 4x - y &= 7 \end{aligned}$

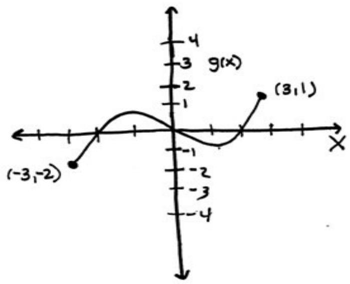
17. $\begin{aligned} x^2 + y &= 6 \\ x + y &= 4 \end{aligned}$

18. $\begin{aligned} x &= 3 - y^2 \\ y &= x - 1 \end{aligned}$

DOMAIN AND RANGE

Domain – All x values for which a function is defined (input values)
Range – Possible y or Output values

EXAMPLE 1



a) Find Domain & Range of g(x).
The domain is the set of inputs of the function.
Input values run along the horizontal axis.
The furthest left input value associated with a pt. on the graph is -3. The furthest right input values associated with a pt. on the graph is 3.
So Domain is [-3,3], that is all reals from -3 to 3.

The range represents the set of output values for the function. Output values run along the vertical axis.
The lowest output value of the function is -2. The highest is 1. So the range is [-2,1], all reals from -2 to 1.

EXAMPLE 2

Find the domain and range of $f(x)=\sqrt{4-x^2}$
Write answers in interval notation.

DOMAIN
For $f(x)$ to be defined $4-x^2 \geq 0$.
This is true when $-2 \leq x \leq 2$
Domain: $[-2,2]$

RANGE
The solution to a square root must always be positive thus $f(x)$ must be greater than or equal to 0.
Range: $[0,\infty)$

Find the domain and range of each function. Write your answer in INTERVAL notation.

19. $f(x)=x^2-5$

20. $f(x)=-\sqrt{x+3}$

21. $f(x)=3\sin x$

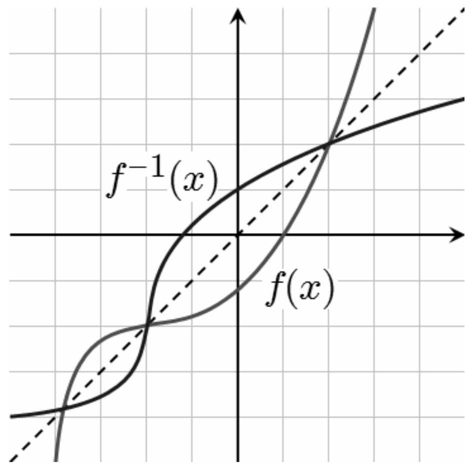
22. $f(x)=\dfrac{2}{x-1}$

INVERSES

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Recall $f^{-1}(x)$ is defined as the inverse of $f(x)$

Example 1:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation



Find the inverse for each function.

23. $f(x)=2x+1$

24. $f(x)=\dfrac{x^2}{3}$

25. $g(x)=\dfrac{5}{x-2}$

26. $y=\sqrt{4-x}+1$

27. If the graph of $f(x)$ has the point (2, 7) then what is one point that will be on the graph of $f^{-1}(x)$?

28. Explain how the graphs of $f(x)$ and $f^{-1}(x)$ compare.

EQUATION OF A LINE

Slope intercept form: $y = mx + b$

Vertical line: x = c (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$
* LEARN! We will use this formula frequently!

Horizontal line: y = c (slope is 0)

Example: Write a linear equation that has a slope of ½ and passes through the point (2, -6)

Slope intercept form

$y = \frac{1}{2}x + b$	Plug in ½ for m
$-6 = \frac{1}{2}(2) + b$	Plug in the given ordered
$b = -7$	Solve for b
$y = \frac{1}{2}x - 7$	

Point-slope form

$y + 6 = \frac{1}{2}(x - 2)$	Plug in all variables
$y = \frac{1}{2}x - 7$	Solve for y

29. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

30. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

31. Use point-slope form to find the equation of the line passing through the point (0, 5) with a slope of 2/3.

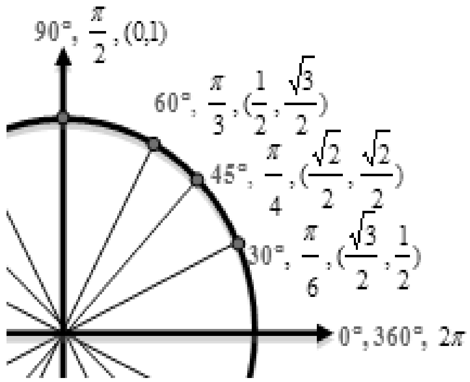
32. Use point-slope form to find a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

33. Use point-slope form to find a line perpendicular to $y = -2x + 9$ passing through the point (4, 7).

34. Find the equation of a line passing through the points (-3, 6) and (1, 2).

35. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3)

UNIT CIRCLE



You can determine the sine or the cosine of any standard angle on the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle. Recall tangent is defined as sin/cos or the slope of the line.

Examples:

$\sin\frac{\pi}{2}=1$ $\quad$ $\cos\frac{\pi}{2}=0$ $\quad$ $\tan\frac{\pi}{2}=und$

***You must have these memorized OR know how to calculate their values without the use of a calculator.**

36. $a.)\ \sin\pi$ $\quad$ $b.)\ \cos\frac{3\pi}{2}$ $\quad$ $c.)\ \sin\left(-\frac{\pi}{2}\right)$ $\quad$ $d.)\ \sin\left(\frac{5\pi}{4}\right)$

$e.)\ \cos\frac{\pi}{4}$ $\quad$ $f.)\ \cos(-\pi)$ $\quad$ g) $\cos\frac{\pi}{3}$ $\quad$ h) $\sin\frac{5\pi}{6}$

i) $\cos\frac{2\pi}{3}$ $\quad$ j) $\tan\frac{\pi}{4}$ $\quad$ k) $\tan\pi$ $\quad$ l) $\tan\frac{\pi}{3}$

m) $\cos\frac{4\pi}{3}$ $\quad$ n) $\sin\frac{11\pi}{6}$ $\quad$ o) $\tan\frac{7\pi}{4}$ $\quad$ p) $\sin\left(-\frac{\pi}{6}\right)$

TRIGONOMETRIC EQUATIONS

Solve each of the equations for $0 \le x < 2\pi$.

37. $\sin x = -\frac{1}{2}$

38. $2\cos x = \sqrt{3}$

39. $4\sin^2 x = 3$

**Recall $\sin^2 x = (\sin x)^2$

**Recall if $x^2 = 25$ then $x = \pm 5$

40. $2\cos^2 x - 1 - \cos x = 0$ *Factor

TRANSFORMATION OF FUNCTIONS

$h(x) = f(x) + c$	Vertical shift c units up	$h(x) = f(x - c)$	Horizontal shift c units right
$h(x) = f(x) - c$	Vertical shift c units down	$h(x) = f(x + c)$	Horizontal shift c units left
$h(x) = -f(x)$	Reflection over the x-axis		

41. Given $f(x) = x^2$ and $g(x) = (x-3)^2 + 1$. How the does the graph of g(x) differ from f(x)?

42. Write an equation for the function that has the shape of $f(x) = x^3$ but moved six units to the left and reflected over the x-axis.

43. If the ordered pair (2, 4) is on the graph of $f(x)$, find one ordered pair that will be on the following functions:

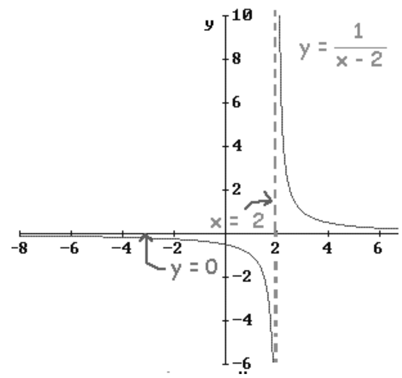
a) $f(x) - 3$ b) $f(x-3)$ c) $2f(x)$ d) $f(x-2) + 1$ e) $-f(x)$

VERTICAL ASYMPTOTES

Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote given the numerator does not equal 0 also (Remember this is called removable discontinuity).
Write a vertical asymptotes as a line in the form x =

Example: Find the vertical asymptote of $y = \dfrac{1}{x-2}$

Since when $x = 2$ the function is in the form 1/0 then the vertical line $x = 2$ is a vertical asymptote of the function.



44. $f(x) = \dfrac{1}{x^2}$

45. $f(x) = \dfrac{x^2}{x^2 - 4}$

46. $f(x) = \dfrac{2 + x}{x^2(1 - x)}$

47. $f(x) = \dfrac{4 - x}{x^2 - 16}$

48. $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

49. $f(x) = \dfrac{5x + 20}{x^2 - 16}$

HORIZONTAL ASYMPTOTES

Determine the horizontal asymptotes using the three cases below.

Case I. Degree of the numerator is less than the degree of the denominator. The asymptote is y = 0.

Example: $y = \dfrac{1}{x-1}$ (As x becomes very large or very negative the value of this function will approach 0). Thus there is a horizontal asymptote at $y = 0$.

Case II. Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Exmaple: $y = \dfrac{2x^2 + x - 1}{3x^2 + 4}$ (As x becomes very large or very negative the value of this function will approach 2/3). Thus there is a horizontal asymptote at $y = \dfrac{2}{3}$.

Case III. Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Example: $y = \dfrac{2x^2 + x - 1}{3x - 3}$ (As x becomes very large the value of the function will continue to increase and as x becomes very negative the value of the function will also become more negative).

Determine all Horizontal Asymptotes.

50. $f(x) = \dfrac{x^2 - 2x + 1}{x^3 + x - 7}$

51. $f(x) = \dfrac{5x^3 - 2x^2 + 8}{4x - 3x^3 + 5}$

52. $f(x) = \dfrac{4x^2}{3x^2 - 7}$

53. $f(x) = \dfrac{(2x-5)^2}{x^2 - x}$

54. $f(x) = \dfrac{-3x+1}{\sqrt{x^2 + x}}$ * Remember $\sqrt{x^2} = \pm x$

This is very important in the use of limits.

EXPONENTIAL FUNCTIONS

Example: Solve for x

$4^{x+1} = \left(\frac{1}{2}\right)^{3x-2}$

$\left(2^2\right)^{x+1} = \left(2^{-1}\right)^{3x-2}$ Get a common base

$2^{2x+2} = 2^{-3x+2}$ Simplify

$2x+2 = -3x+2$ Set exponents equal

$x = 0$ Solve for x

Solve for x:

55. $3^{3x+5} = 9^{2x+1}$

56. $\left(\frac{1}{9}\right)^x = 27^{2x+4}$

57. $\left(\frac{1}{6}\right)^x = 216$

LOGARITHMS

The statement $y = b^x$ can be written as $x = \log_b y$. They mean the same thing.

REMEMBER: A LOGARITHM IS AN EXPONENT

Recall $\ln x = \log_e x$

The value of e is 2.718281828… or $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$

Example: Evaluate the following logarithms

$\log_2 8 = ?$

In exponential for this is $2^? = 8$

Therefore $? = 3$

Thus $\log_2 8 = 3$

Evaluate the following logarithms

58. $\log_7 7$

59. $\log_3 27$

60. $\log_2 \frac{1}{32}$

61. $\log_{25} 5$

62. $\log_9 1$

63. $\log_4 8$

64. $\ln\sqrt{e}$

65. $\ln\frac{1}{e}$

PROPERTIES OF LOGARITHMS

$\log_b xy = \log_b x + \log_b y$ $\qquad \log_b \dfrac{x}{y} = \log_b x - \log_b y$ $\qquad \log_b x^y = y \log_b x$ $\qquad b^{\log_b x} = x$

Examples:

Expand $\log_4 16x$	Condense $\ln y - 2\ln R$	Expand $\log_2 7x^5$
$\log_4 16 + \log_4 x$	$\ln y - \ln R^2$	$\log_2 7 + \log_2 x^5$
$2 + \log_4 x$	$\ln \dfrac{y}{R^2}$	$\log_2 7 + 5\log_2 x$

Use the properties of logarithms to evaluate the following

66. $\log_2 2^5$

67. $\ln e^3$

68. $\log_2 8^3$

69. $\log_3 \sqrt[5]{9}$

70. $2^{\log_2 10}$

71. $e^{\ln 8}$

72. $9\ln e^2$

73. $\log_9 9^3$

74. $\log_{10} 25 + \log_{10} 4$

75. $\log_2 40 - \log_2 5$

76. $\log_2 \left(\sqrt{2}\right)^5$

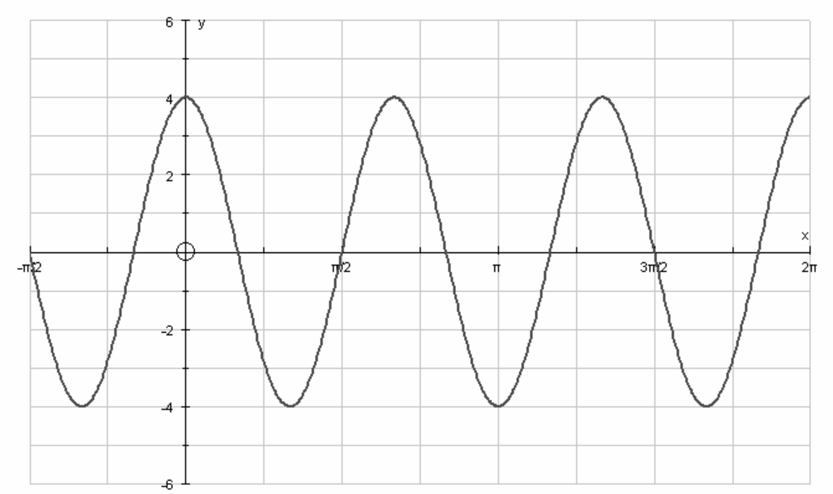
EVEN AND ODD FUNCTIONS

Recall:
Even functions *are functions that are symmetric over the y-axis.*
To determine algebraically we find out if $f(x) = f(-x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected across the y-axis*)*

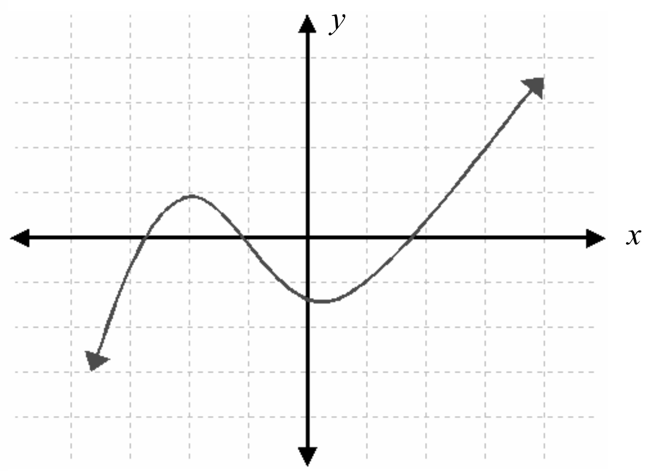
Odd functions *are functions that are symmetric about the origin.*
To determind algebraically we find out if $f(-x) = -f(x)$
*(*Think about it what happens to the coordinate* $(x, f(x))$ *when reflected over the origin*)*

State whether the following graphs are even, odd or neither, show ALL work.

77. ______________________



78. ______________________



79. ______________________
$f(x) = 2x^4 - 5x^2$

80. ______________________
$g(x) = x^5 - 3x^3 + x$

81. ______________________
$h(x) = 2x^2 - 5x + 3$

82. ______________________
$j(x) = 2\cos x$

83. ______________________
$k(x) = \sin x + 4$

84. ______________________
$l(x) = \cos x - 3$

You may need to type in the following URL by hand instead of clinking on the link on the next page. There is a dash, or " - " between *calculus* and *help.* I had difficulty when I clicked the link on the next page. Try clicking on the link on this page.

Here is the URL (CLICK on this link)

http:/www.calculus-help.com/tutorials

LIMITS!

First of all, watch some explanations of limits online. Go to the following website http://www.calculus-help.com/tutorials and watch lessons 1-5. I've included some questions below that go with each lesson. Then answer the corresponding questions after watching each lesson.

Lesson 1: What is a limit?

1. How would you describe a limit?

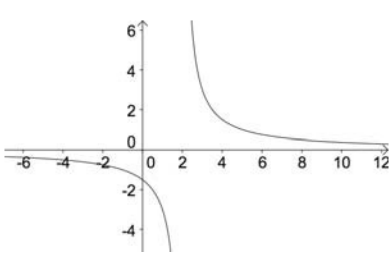
2. Some graphs are straightforward, like $f(x) = x^2$. What's $\lim_{x\to 3} x^2$?

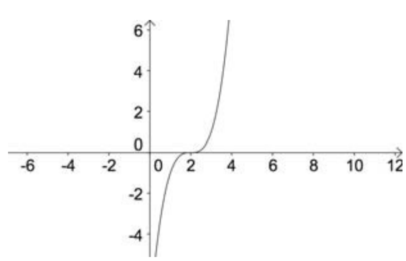
3. Some graphs are more 'mysterious', like $f(x) = \dfrac{x^2+3x-4}{x-1}$. What's $\lim_{x\to 1} \dfrac{x^2+3x-4}{x-1}$?
   (Either find the limit, if you know how, or describe how to find it based on the explanation you saw online)

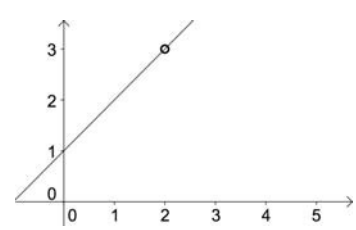
Lesson 2: When does a limit exist?

4. How is a limit like two friends meeting at a diner?

5. Look at the following graphs. Which one(s) have a limit that exists at x=2, and which one(s) don't have a limit that exists at x=2?

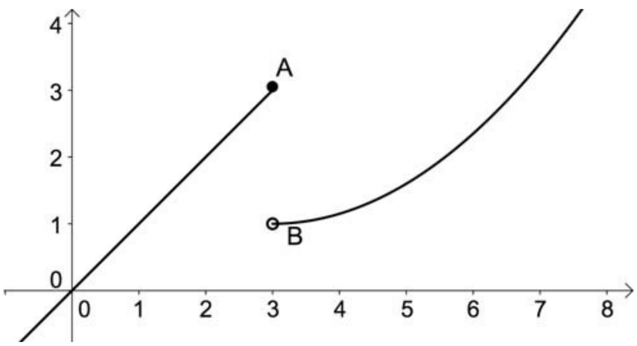






6. What is meant by a "right hand limit" and a "left hand limit"?

7. For the following graph, find: $\lim_{x \to 3^+} f(x) =$ $\lim_{x \to 3^-} f(x) =$



8. For a limit to exist, what has to be true for the left hand and the right hand limits?

Lesson 3: How do you evaluate limits?

9. What are the 3 methods for evaluating limits?

10. When can you use the substitution method?

11. When can you use the factoring method?

12. When can you use the conjugate method?

13. Figure out which method to use for the following limits, and evaluate them:

a. $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} =$

b. $\lim_{x \to 1} \frac{4x + 5}{6x - 1} =$

c. $\lim_{x \to 1} \frac{x^2 + 3x - 4}{x - 1} =$

Lesson 4: Limits and Infinity

14. How do you know if a function has a vertical asymptote?

15. When you take the limit of a function at its vertical asymptote, the limit will be _____ or ______.

16. To determine if a function has a horizontal asymptote, look at the…

17. If the degrees of the numerator and denominator are equal, how do you find the horizontal asymptote?

18. If the degree of the denominator is greater than that of the numerator, what's the horizontal asymptote?

19. If the degree of the denominator is less than that of the numerator, what's the horizontal asymptote?

20. If we say that the limit of a function EQUALS INFINITY, this really means that….

Lesson 5: Continuity

21. What does it mean for a function to be continuous?

22. What are the 3 types of discontinuity? Draw an example of a graph of each kind below:

23. In order to be continuous, 3 things must be true:
    - There must be no _______________
    - There must be no _______________
    - The limit must be equal to the ____________________________________

24. An "easy" way to tell if a function is continuous is this: